Shortest Path

Find the shortest path from node 1 to node m. (Where m is the terminal node.)

Let (i, j) be an arc from node i to node j

Decision Variables Xij = 1 if (i, j) is on the shortest path, and 0 otherwise

Data: Cij = cost to traverse arc (i, j)

*Think of shipping 1 unit from node 1 to node m.*

Formulation:

Objective Minimize the Distance (Cost) that the 1 unit has to travel from node 1 to node m

Min

Constraints:

The first node will have a 1 unit output. The intermediate nodes will have 1 in and 1 out (equilibrium). The terminal node (node m) will have a 1 unit input.

Xij = binary (must be 0 or 1)

However, the A matrix (constraint matrix) is totally unimodular. If b (right hand side values) are all integers (and they are, since they are 1, -1, and 0). Therefore, we know the solution will be integer. We can replace the binary constraints with the linear programming equivalents:

0 <= Xij <= 1 and we will still get an integer solution.

**If Cij >= 0, then Xij only needs the non-negativity restrictions.**

We will have a binary solution – but can treat the problem as a linear program with non-negativity constraints on Xij. And the flow constraints listed above.

Dual:

Max: W1 - Wm

Constraints:

Wi – Wj <= Cij (for all i,j pairs)

Wi’s are unrestricted

*The Wi’s are variables for the nodes.*

Algorithms are set to work for non-negativity